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ANALYSIS OF ROTATING, RECIRCULATING TURBULENT FLOW AND HEAT TRANSFER IN AN ENCLOSURE WITH FLUID THROUGHFLOW

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INTRODUCTION

PROBLEMS of flow and heat transfer in enclosures with one or more rotating walls and with coolant throughflow are encountered in gas turbines and other rotating devices. Such enclosures and cavities are situated adjacent to the rotating shaft and are found in a variety of sizes and shapes. Owing to the broad range of possible geometries and operating conditions, it is impractical to consider performing experiments for even a modest fraction of all the configurations of interest. It is, therefore, appropriate to seek analytical predictions for the fluid flow and heat-transfer characteristics.

To reflect reality, an analytical model for the flow in the aforementioned enclosures should take account of both turbulence and recirculation. From an examination of the relevant literature, it appears that the only published analyses are limited either to laminar flow or to boundary layer type turbulent flow (i.e. without recirculation), as exemplified by [1, 2].

In considering a next step forward, it appeared logical to the present authors to examine what can be done with the rotating cavity—fluid throughflow problem by employing a relatively simple turbulence model. More sophisticated models have been formulated [3] involving additional conservation equations for predicting the turbulent transport characteristics, subject to input values of experimentally determined parameters. It is felt, however, that application of these models to the present class of problems is a later step.

The present analysis is formulated to permit recirculation, and the turbulence model takes account of differences in the transport processes near the walls and in the core. An outline of the analytical formulation will be given here along with representative results. Comparisons are made with available experimental data [4] in order to help assess the efficacy of the approach. The details of the work are given in [5].

A schematic diagram of the problem to be analyzed is presented in Fig. 1, which shows a cylindrical cavity with a coolant fluid entering and leaving through central apertures in the respective disks that bound the enclosure. The entering flow is turbulent. The left-hand disk is rotating, as is the attached inlet pipe, whereas the other walls are stationary. The thermal boundary conditions are stated in the diagram, along with dimensional nomenclature. This problem was selected for analysis in order to match the experimental set-up of [4].

OUTLINE OF THE ANALYSIS AND SOLUTION

The starting point of the analysis was the four conservation equations for r, ϕ and z momentum and continuity, in which the three velocity components and the pressure appear as unknowns. The flow field was assumed to be axisymmetric, and this facilitated the elimination of the pressure in terms of the vorticity ω. Further manipulation produced a set of three coupled partial differential equations for ω/r, the stream function ψ, and rv_ϕ.

The flow field was subdivided into two regions: a wall region and a core. The aforementioned governing differential equations were employed both in the core and on the boundary between the wall and core regions. For the core, the shear stress terms were evaluated using a uniform, isotropic eddy diffusivity characterized by ε/ν.

For the wall region, the so-called wall function [6] was employed. It was obtained from numerical integration of a simplified momentum equation which balances the shear, pressure, and centrifugal forces. The turbulence model used for the wall region is the Prandtl mixing length modified by the Van Driest damping factor. The wall function is an algebraic fit of the numerical solution of the simplified momentum equation, and it inter-relates velocity, distance from the wall, and wall shear. This relationship contains the pressure and centrifugal forces as input paremeters. The wall function expression given in [6] was adapted to take account of these input parameters as they occur in the present problem.

The matching of the wall and core regions was performed in such a way that all the conservation laws were satisfied. For the flow field adjacent to the disks, the boundary between the regions was at z = constant, whereas adjacent to the shroud the matching was at r = constant.

Owing to the assumption of constant thermophysical properties, the flow field is independent of the temperature field. Therefore, the energy equation can be solved after the solution for the velocity field has been obtained. The molecular and turbulent Prandtl numbers, which appear as parameters in the energy equation, were both assigned values of unity. The reason for using a molecular Prandtl number of unity rather than 0·7 was that it enabled certain velocity-field wall function results to be employed in solving the energy equation for the wall region.

A finite difference scheme was employed for the solutions, based on the contrived transient—explicit method described in [7]. For this purpose, the enclosure was subdivided by a grid containing 13 × 15 elements (axial × radial).

The positioning of the boundary between the core and wall regions was checked after each solution. With the aid of the wall functions, the dimensionless eddy diffusivity ε/ν was evaluated at each grid point on the boundary line. A comparison of the average of these values with that for the core flow was used as a criterion as to whether the boundary line had to be shifted and the calculation repeated.

RESULTS AND CONCLUSIONS

Local Nusselt number results for three cases which permit comparison with experiment are shown in Fig. 2. For each case, the results for the rotating disk and the stationary shroud are respectively given in the left- and right-hand graphs. The dimensionless parameters are defined as: $Nu = hr_0/k$, $Re_i = \bar{v}_{zi}(2r_i)/\nu$, $Re_\Omega = r_0^2\Omega/\nu$. The local heat-transfer coefficient h is the quotient of the local heat flux

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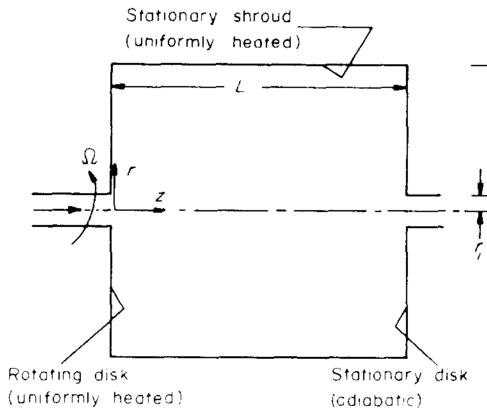

FIG. 1. Schematic diagram of the enclosure.

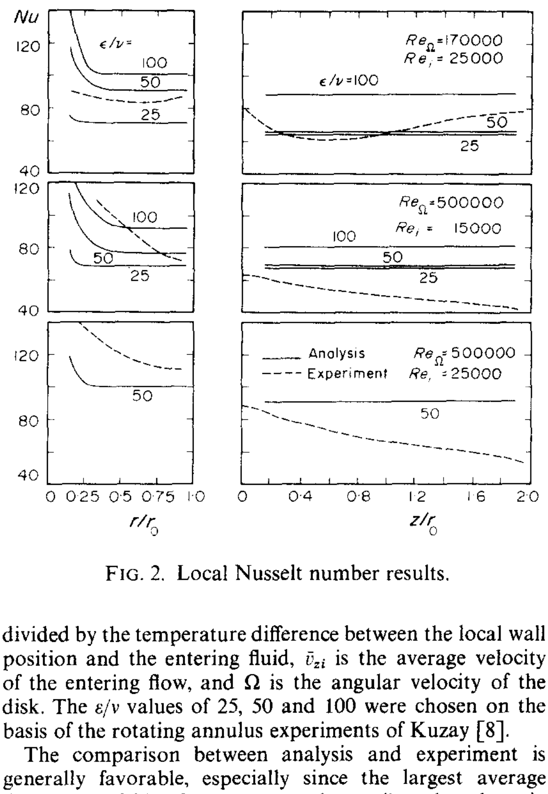

FIG. 2. Local Nusselt number results.

divided by the temperature difference between the local wall position and the entering fluid, $\bar{v}_{zi}$ is the average velocity of the entering flow, and Ω is the angular velocity of the disk. The ε/ν values of 25, 50 and 100 were chosen on the basis of the rotating annulus experiments of Kuzay [8].

The comparison between analysis and experiment is generally favorable, especially since the largest average deviations of 20–25 per cent can be attributed, at least in part, to the uncertainties of the experimental data. On the other hand, there are a number of indications that a model which assumes a uniform, isotropic turbulent diffusivity in the core is not fully adequate. In particular, the spatial variations of the analytical and experimental results are, in some cases, not consonant. In fact, aside from the range of smaller radial positions on the rotating disk, the analytical results are essentially independent of position on the respective disk and shroud surfaces. Furthermore, the relationship between the analytical and experimental results is different on the disk and on the shroud.

On the basis of the foregoing, it appears that the model used here should be adequate for preliminary design calculations but that a more refined model might well be considered for a detailed final design.

A sequence of graphs will now be presented to provide insight into the flow field. These results correspond to the Re_i and Re_Ω of the uppermost case of Fig. 2, with $\varepsilon/\nu = 50$. Figure 3 shows the streamlines in the r, z plane. The stream

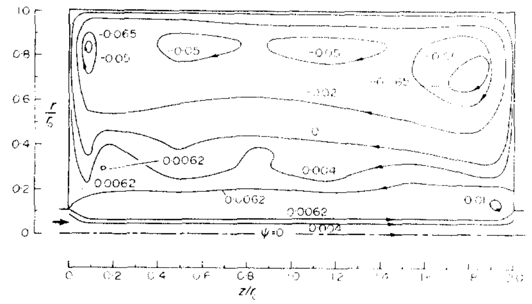

FIG. 3. Streamlines in r, z plane. $Re_\Omega = 170\,000$, $Re_i = 25\,000$, $\varepsilon/\nu = 50$.

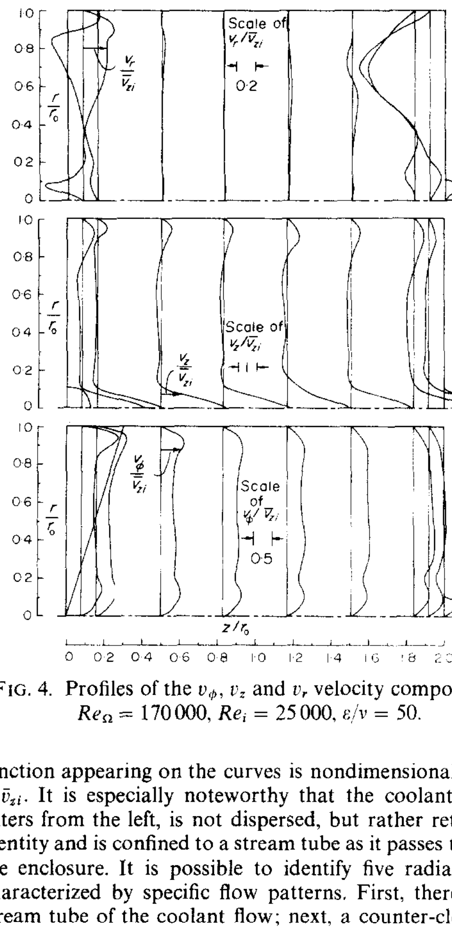

FIG. 4. Profiles of the v_ϕ, v_z and v_r velocity components. $Re_\Omega = 170\,000$, $Re_i = 25\,000$, $\varepsilon/\nu = 50$.

function appearing on the curves is nondimensionalized by $r_0^2\bar{v}_{zi}$. It is especially noteworthy that the coolant, which enters from the left, is not dispersed, but rather retains its identity and is confined to a stream tube as it passes through the enclosure. It is possible to identify five radial bands characterized by specific flow patterns. First, there is the stream tube of the coolant flow; next, a counter-clockwise vortex zone; beyond that, an axial backflow region; then, a clockwise vortex zone; and, finally, an axial forward flow region near the wall. In view of the complexity of such a flow field, unsteadiness is expected to exist in practice.

Figure 4 is an array of three graphs which respectively portray profiles of v_ϕ, v_z and v_r. The profiles at various axial stations are plotted as a function of the radial coordinate. The scale of the dimensionless velocity variable is shown in each graph.

The v_ϕ profiles, all of which correspond to axial stations outside the boundary layers on the disks, are very nearly identical all across the enclosure. The profiles of the axial velocity v_z reveal the presence of three regions distributed across the radius of the cavity. The inner region is characterized by a forward flow through the central stream tube, the middle region contains a backflow, whereas the outer region corresponds to a forward flow along the shroud. The radial velocities are generally small except near the rotating and stationary disks.

A fuller presentation of results is available in [5].

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